

Neutrino Masses and Bimaximal Mixing*

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Abstract

Solar and atmospheric neutrino anomalies are described by bimaximal mixing of three Majorana neutrinos. Neutrino oscillations in appearance $\nu_e \Rightarrow \nu_\mu, \nu_\tau$ and in disappearance ν_e long baseline and atmospheric experiments are sensitive to deviations from the ideal bimaximal mixing. It is suggested that these deviations may be dominated by a rotation in the electron–muon plane of the generation space. Simple seesaw models are described supporting this idea. The rotation angle is estimated from Fritzsch’s relation. Predictions are presented for oscillations in long baseline experiments, for solar neutrinos, and for the rates of neutrinoless double beta decays.

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1 Introduction

Neutrino masses and oscillations[1, 2] belong to the most exciting area of particle physics. It is exactly this area where the Standard Model of quarks and leptons is challenged in a most successful way. The recent discovery of muon neutrino oscillations[3] and the evidence for solar neutrino deficits[4] in comparison to the Standard Solar Model[5] demonstrate that the neutrinos are massive. Phenomenological analyses favor two solutions of the solar and atmospheric neutrino problems, see e.g.[6]–[13] and for a recent review see [14]:

- large $\nu_\mu - \nu_\tau$ mixing for atmospheric anomalies and matter enhanced[15] small mixing angle oscillations (SMA MSW) for solar neutrinos, see e.g.[16].
- vacuum oscillations and bimaximal[17]–[26] or nearly bimaximal[27] mixing of three light neutrinos.

We believe that one of these two possibilities is true although other options are not completely excluded, see e.g.[7, 10, 14, 28]. In a near future experimental information from SuperKamiokande and SNO will select the best scenario. In fact a recent measurement of day–night asymmetries[29] imposes constraints on SMA MSW solutions. On the other hand the vacuum oscillation solutions may be confirmed or excluded by final results on seasonal effects and recoil energy distributions, see [11, 13, 30] and references therein. A preliminary data from SuperKamiokande[31] are analyzed in Ref.[13].

In this article we discuss some consequences of the bimaximal mixing scenario. Evidently solutions of ‘just so’ type[32] are preferred by the existing data. However we do not exclude a possibility[18] that a larger range of Δm_\odot^2 is allowed for solar neutrinos if errors are underestimated in one of the radiochemical measurements. In such a case ‘just so’ fits are also better[11, 13]. In the following discussion we use as input parameters[33]

$$\Delta m_\odot^2 = 4.3 \times 10^{-10} \text{ eV}^2$$

for oscillations of solar and

$$1.5 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{atm}^2 \leq 6 \times 10^{-3} \text{ eV}^2$$

for oscillations of atmospheric neutrinos.

2 Maki–Nakagawa–Sakata lepton mixing matrix

The Dirac masses of quarks and charged leptons exhibit a strongly ordered hierarchical structure

$$m_u \ll m_c \ll m_t, \quad m_d \ll m_s \ll m_b, \quad m_e \ll m_\mu \ll m_\tau.$$

We assume a similar hierarchical structure for the Dirac masses of neutrinos. However this hierarchical structure is drastically modified by huge Majorana masses of righthanded neutrinos. As a result of the seesaw mechanism[34] three light and nearly lefthanded neutrinos appear in the mass spectrum. Their Majorana masses can form patterns very different from the hierarchical one. For the same reason the structures of quark and lepton mixing matrices are

also quite different. For quarks the Cabibbo-Kobayashi-Maskawa matrix[35] V_{CKM} is nearly diagonal and its largest off-diagonal elements

$$|V_{us}| \simeq |V_{cd}| \approx \theta_c \approx 0.22 \quad (1)$$

are fairly small. All other off-diagonal elements of V_{CKM} are much smaller and can be parameterized by higher powers of the Cabibbo angle θ_c [36]. V_{CKM} can be written as a product of two matrices describing unitary transformations of quarks with weak isospin projections $I_3 = +1/2$ (u, c, t) and $I_3 = -1/2$ (d, s, b) respectively:

$$V_{CKM} = V_+^\dagger V_- \quad (2)$$

In the Standard Model only V_{CKM} is observable. In models of quark masses and mixing both matrices V_\pm are specified and related to Yukawa couplings of up and down type quarks. A particularly interesting approach[37] relates angles in V_+ and V_- to mass ratios of up and down type quarks respectively. In this way a relation is derived for the mixing between the first two generations

$$\theta_c = \left| \sqrt{m_d/m_s} + e^{i\phi} \sqrt{m_u/m_c} \right| \approx \sqrt{m_d/m_s} \quad (3)$$

The mixing angle θ_c is dominated by the contribution from the $I_3 = -1/2$ sector.

The flavor mixing matrix[2] for three light neutrinos relates the neutrino mass and flavor eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = U_{MNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (4)$$

U_{MNS} can be written as a product of two matrices U_\pm describing transformations of the neutrinos ($I_3 = +1/2$) and the charged leptons ($I_3 = -1/2$):

$$U_{MNS} = U_-^\dagger U_+ \quad (5)$$

It is plausible that the pattern of mixing angles in U_- describing the charged lepton sector resembles that in V_- for the down type quarks, i.e. there is an appreciable mixing in the electron-muon plane whereas the other mixing angles are much smaller

$$U_- \approx \begin{pmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (6)$$

where[37]

$$|s| \simeq \sqrt{m_e/m_\mu} \simeq \theta_c/3 \simeq 0.07 \quad \text{and} \quad c = \sqrt{1-s^2} \approx 1 - s^2/2 \quad (7)$$

The second equality in (7) follows from grand unification relations[38]

$$m_d \simeq 3m_e \quad , \quad m_s \simeq m_\mu/3 \quad (8)$$

In the sector $I_3 = +1/2$ the situation is quite different. The structure of the matrix U_+ is not directly related to the Dirac mass matrix \mathbf{m}_D for neutrinos. It is strongly affected by the structure of the Majorana mass matrix \mathbf{M}_R for the righthanded neutrinos. As a consequence

the structure of U_+ is different from the structure of V_+ . We assume that the mixing of solar and atmospheric neutrinos is close to bimaximal,

$$U_+ \approx U_{bm} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}. \quad (9)$$

We also assume that the dominant contribution to deviations from the ideal bimaximal mixing originates from the sector $I_3 = -1/2$, i.e. from the form of the matrix U_- . (Later in this paper we will demonstrate simple seesaw models in which corrections to eq.(9) are fairly small.) The main goal of the present paper is to describe phenomenological consequences of these two assumptions. For the sake of simplicity we neglect CP violation and consider all elements of U_{MNS} as real. Then the lepton mixing matrix is completely specified. From eqs.(5), (6) and (9) we obtain

$$U_{MNS} \approx \begin{pmatrix} c/\sqrt{2} - s/2, & c/\sqrt{2} + s/2, & s/\sqrt{2} \\ -c/2 - s/\sqrt{2}, & c/2 - s/\sqrt{2}, & c/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} \quad (10)$$

with s and c given in eq.(7).

3 Neutrino oscillations

It is well known[8, 12] that for $\Delta m_\odot^2 \ll \Delta m_{atm}^2$ and small $|U_{e3}|$, oscillations of solar neutrinos effectively decouple from oscillations in long baseline (LBL) experiments and those of atmospheric neutrinos. Both types of oscillations can be described by effective two-neutrino oscillations characterized by phase differences

$$\Delta_\odot = \frac{\Delta m_\odot^2 L}{4E} \quad (11)$$

for solar, and

$$\Delta_{atm} = \frac{\Delta m_{atm}^2 L}{4E} \quad (12)$$

for LBL and atmospheric oscillations. In the above equations L is the distance traveled by a neutrino of energy E . Probabilities of transitions between different neutrino flavors in atmospheric and LBL experiments are given by, see [14] and references therein:

$$\begin{aligned} \mathcal{P}_{LBL}(\nu_\alpha \rightleftharpoons \nu_\beta) &= A_{\alpha\beta} \sin^2 \Delta_{atm} \quad (\alpha \neq \beta) \\ \mathcal{P}_{LBL}(\nu_\alpha \rightarrow \nu_\alpha) &= 1 - B_\alpha \sin^2 \Delta_{atm} \quad , \end{aligned} \quad (13)$$

where $(\alpha, \beta = e, \mu, \tau)$

$$\begin{aligned} A_{\alpha\beta} &= 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \quad , \\ B_\alpha &= 4|U_{\alpha 3}|^2 (1 - |U_{\alpha 3}|^2) \quad . \end{aligned} \quad (14)$$

Then eq.(10) implies

$$\begin{aligned} A_{e\mu} &= s^2 c^2, & A_{e\tau} &= s^2, & A_{\mu\tau} &= c^2, \\ B_e &= 1 - c^4, & B_\mu &= 1 - s^4, & B_\tau &= 1, \end{aligned} \quad (15)$$

and using (7) we estimate that for appearance type $\nu_e \rightleftharpoons \nu_\mu$ and $\nu_e \rightleftharpoons \nu_\tau$ LBL experiments

$$\sin^2 2\vartheta_a \approx 0.005 \quad (16)$$

whereas

$$\sin^2 2\vartheta_d \approx 0.01 \quad (17)$$

for disappearance of ν_e and $\bar{\nu}_e$ neutrinos. These predictions can be tested in future high precision experiments. However the numbers are rather small. The estimation (16) is about two times smaller than planned sensitivity of MINOS[39] and eq.(17) predicts disappearance of electron neutrinos at the level 20 times smaller than the present limit from CHOOZ[40]. Still these estimations are larger than those following from SMA MSW solutions for which $\sin^2 2\vartheta_d$ is driven to much smaller values by the recent data on day-night asymmetries for solar neutrinos[29].

For solar neutrinos, c.f.[14] and references therein,

$$\mathcal{P}_\odot(\nu_e \rightarrow \nu_e) = \left(1 - |U_{e3}|^2\right) \left(1 - \sin^2 2\vartheta_\odot \sin^2 \Delta_\odot\right) + |U_{e3}|^4 \quad , \quad (18)$$

where

$$\sin^2 2\vartheta_\odot = 4|U_{e1}|^2 |U_{e2}|^2 / \left(1 - |U_{e3}|^2\right)^2 \quad . \quad (19)$$

For the mixing matrix (10)

$$\mathcal{P}_\odot = \left(1 - s^2\right) \left(1 - \sin^2 2\vartheta_\odot \sin^2 \Delta_\odot\right) + \mathcal{O}(s^4) \quad , \quad (20)$$

with

$$\sin^2 2\vartheta_\odot \approx 1 - s^2 \approx 0.99 \quad . \quad (21)$$

It is evident that the simple picture proposed in this paper can be falsified when $\sin^2 2\vartheta_\odot$ derived from solar neutrino data is significantly below 1. In Ref.[7] the best fit to vacuum oscillations is

$$\Delta m_\odot^2 = 6.5 \times 10^{-11} \text{ eV}^2, \quad \sin^2 2\vartheta_\odot = 0.75 \quad (22)$$

which seems to be in conflict with the estimation in (21). In Ref.[13], see also [11], a new preliminary data from SuperKamiokande [33] are included into analysis leading to acceptable solutions in three regions (called *A*, *C* and *D*) of parameter space

$$\begin{array}{lll} C : & \Delta m_\odot^2 = 4.4 \times 10^{-10} \text{ eV}^2, & \sin^2 2\vartheta_\odot = 0.93, & \text{gof} = 14\% \\ D : & \Delta m_\odot^2 = 6.4 \times 10^{-10} \text{ eV}^2, & \sin^2 2\vartheta_\odot = 1.00, & \text{gof} = 8\% \\ A : & \Delta m_\odot^2 = 6.5 \times 10^{-11} \text{ eV}^2, & \sin^2 2\vartheta_\odot = 0.70, & \text{gof} = 6\% \end{array} \quad (23)$$

In eq.(23) the parameter *gof* (goodness-of-fit) is the probability that a random repeat of the given experiment would observe a greater χ^2 , assuming the model is correct. The region *A* is the same as selected by the fits of Ref.[7]. Clearly the analysis of Ref.[13] gives results in better agreement with eq.(21). It is also seen that new data on solar neutrinos will seriously test the estimations presented in this article.

4 Neutrinoless double beta decay

Let us discuss now predictions for neutrinoless double beta decays $(0\nu 2\beta)^1$. Probabilities of such transitions depend on a mass parameter B which is equal to the absolute value of the element $(N_A)_{11}$ of the matrix

$$N_A = U_{MNS} M_{L,A} U_{MNS}^T \quad . \quad (24)$$

$M_{L,A}$ (for $A = I, II, III, IV$) denotes a diagonal matrix of Majorana masses for the three light neutrinos:

$$M_{L,A} = m \, \text{diag}(\lambda_1, \lambda_2, \lambda_3) \quad , \quad (25)$$

where $|\lambda_i| \leq 1 + \mathcal{O}(\epsilon, \xi)$; ϵ and ξ are small parameters to be discussed in the following section. In our convention the eigenvalues λ_i are ordered in such a way that

$$\Delta m_\odot^2/m^2 = |\lambda_1^2 - \lambda_2^2| \quad (26)$$

and

$$\Delta m_{atm}^2/m^2 \approx |\lambda_1^2 - \lambda_3^2| \approx |\lambda_2^2 - \lambda_3^2| \quad . \quad (27)$$

For bimaximal mixing ν_e is an equal mixture of the mass eigenstates 1 and 2.

Considering $\Delta m_\odot^2/\Delta m_{atm}^2$ as a small perturbation one obtains $|\lambda_1| = |\lambda_2|$. Then, up to irrelevant sign redefinitions there are only four mass patterns consistent with the data on $0\nu 2\beta$:

- I) $m = \sqrt{\Delta m_{atm}^2} \quad , \quad \lambda_1 = \lambda_2 = 0, \lambda_3 = 1$
- II) $m = \sqrt{\Delta m_{atm}^2} \quad , \quad \lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 0$
- III) $m = \sqrt{\Delta m_{atm}^2} \quad , \quad \lambda_1 = \lambda_2 = 1, \lambda_3 = 0$
- IV) $m = \mathcal{O}(1 \text{ eV}) \gg \sqrt{\Delta m_{atm}^2} \quad , \quad \lambda_1 = -1, \lambda_2 = 1, \lambda_3 = \eta \text{ where } \eta = \pm 1 \quad .$

In case *IV* $\Delta m_{atm}^2/m^2$ is considered as a small perturbation². The states 1 and 2 have opposite CP parities (i.e. λ_1 and λ_2 have opposite signs, see e.g.[14]) due to the experimental bound[42]:

$$B < 0.2 \text{ eV} \quad (28)$$

which is violated for $m > 0.2 \text{ eV}$ and $\lambda_1 = \lambda_2 = 1$. The argument is analogous to the case *III* which is considered in the following, c.f. eq.(35).

If corrections to bimaximal mixing are neglected the following mass matrices are obtained

$$\bar{N}_I = \frac{1}{2} \sqrt{\Delta m_{atm}^2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad , \quad (29)$$

¹A model independent analysis is presented in a recent preprint[41].

²The case $\Delta m_{atm}^2/m^2 = \mathcal{O}(1)$ requires much more tuning of parameters than the case *IV*. We do not discuss this case in the present article because it is not clear if a reasonable model of this kind exists. Furthermore an original motivation for degenerate neutrino masses is an appreciable neutrino contribution to dark matter in the Universe whereas this contribution is small if $\Delta m_{atm}^2/m^2$ is not small.

$$\bar{N}_{II} = \frac{1}{\sqrt{2}} \sqrt{\Delta m_{atm}^2} \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} , \quad (30)$$

$$\bar{N}_{III} = \frac{1}{2} \sqrt{\Delta m_{atm}^2} \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} , \quad (31)$$

$$\bar{N}_{IV} = \frac{1}{2} m \begin{pmatrix} 0 & \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \eta & \eta \\ -\sqrt{2} & \eta & \eta \end{pmatrix} , \quad (32)$$

where for $A = I, II, III, IV$:

$$\bar{N}_A = U_{bm} M_{L,A} U_{bm}^T . \quad (33)$$

In the following section we consider also matrices

$$\widetilde{N}_A = U_+ M_{L,A} U_+^T \quad (34)$$

which include deviations from the ideal bimaximal mixing due to the sector $I_3 = +1/2$. The textures (29)–(32) have been discussed in the literature[19]–[24]. It is seen that only \bar{N}_{III} leads to non-zero rates of $0\nu 2\beta$ decays. In this case deviations from bimaximal mixing can be neglected and one obtains the following prediction

$$|N_{III}|_{11} \approx \sqrt{\Delta m_{atm}^2} = (6 \pm 2) \times 10^{-2} \text{ eV} . \quad (35)$$

In the three other cases the role of U_- is essential. Keeping only leading terms one obtains

$$|N_I|_{11} \approx \frac{1}{2} s^2 \sqrt{\Delta m_{atm}^2} = (1.5 \pm 0.5) \times 10^{-4} \text{ eV} , \quad (36)$$

$$|N_{II}|_{11} \approx \sqrt{2} |s| \sqrt{\Delta m_{atm}^2} = (6 \pm 2) \times 10^{-3} \text{ eV} , \quad (37)$$

$$|N_{IV}|_{11} \approx \sqrt{2} |s| m \approx 0.1 m . \quad (38)$$

Evidently the cases *III* and *IV* can be confirmed or ruled out by next generation experiments which may be sensitive to B as low as 0.01 eV[43]. Even the present limit[42] implies that for degenerate masses m must be smaller than 2 eV.

5 Patterns of neutrino masses

In this section effects are considered due to non-zero value of the ratio $\Delta m_\odot^2 / \Delta m_{atm}^2$ which were neglected in the discussion of the preceding section. We present simple seesaw models which show that these corrections are quite small, so the estimations (35)–(38) are not much affected. In the following we consider the four cases *I–IV* corresponding to different neutrino mass patterns.

5.1 Case I

A class of seesaw models corresponding to case *I* has been described in[19]. In these models the third mass eigenstate is much heavier than the other two whose masses are fairly close. In the

following this mass pattern ($m_1 \approx m_2 \ll m_3$) is called *semi-hierarchical*. In terms of a small parameter

$$|\epsilon| = \left(\frac{\Delta m_\odot^2}{2\Delta m_{atm}^2} \right)^{1/3} \approx 0.004 \quad (39)$$

the eigenvalues λ_i are given by the expansions:

$$\begin{aligned} \lambda_1 &= \epsilon - \epsilon^2/2 + \dots \\ \lambda_2 &= -\epsilon - \epsilon^2/2 + \dots \\ \lambda_3 &= 1 + \epsilon^2 + \dots \end{aligned} \quad (40)$$

For semi-hierarchical mass pattern ν_e is a linear combination of the two lighter mass eigenstates of opposite CP parities. The element $(\widetilde{N}_I)_{11}$ of the matrix

$$\widetilde{N}_I = U_+ M_{L,I} U_+^T \quad (41)$$

is strongly suppressed and the main contribution to $0\nu 2\beta$ transitions originates from the mixing in the sector $I_3 = -1/2$. The estimation of $|N_I|_{11}$ given in eq.(36) is not affected as can be seen from the explicit form of the matrix U_+ :³

$$U_+ = \begin{pmatrix} (1 + \epsilon/2)/\sqrt{2} & (1 - \epsilon/2)/\sqrt{2} & \epsilon \\ -1/2 - \epsilon & 1/2 - \epsilon & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix} + \mathcal{O}(\epsilon^2) = U_{bm} + \mathcal{O}(\epsilon) \quad (42)$$

and

$$\widetilde{N}_I = \frac{1}{2} \begin{pmatrix} 0 & 0 & 2^{3/2}\epsilon \\ 0 & 1 & 1 \\ 2^{3/2}\epsilon & 1 & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2) \quad (43)$$

In case I a strongly ordered *hierarchical* mass pattern ($m_1 \ll m_2 \ll m_3$) is obtained if e.g.

$$\lambda_1 = \epsilon^2 + \dots, \quad \lambda_2 = \epsilon + \dots, \quad \lambda_3 = 1 + \dots, \quad (44)$$

with

$$|\epsilon| = \sqrt{\Delta m_\odot^2 / \Delta m_{atm}^2} \approx 3.7 \times 10^{-4} \quad (45)$$

The magnitude of ϵ is an order of magnitude smaller than for semi-hierarchical spectrum. There is no suppression of $(N_I)_{11}$ because the lighter states have the same CP parity but the parameter ϵ is small. Seesaw models leading to hierarchical spectrum may require some fine tuning. Thus we believe that hierarchical mass pattern is less attractive from theoretical point of view. In spite of theoretical prejudices we note that for both patterns predictions for oscillations and $0\nu 2\beta$ decays are similar.

³ In notation of Ref.[19] $r = 2^{3/2}\epsilon$. The columns of the matrix U_+ in eq.(42) are normalized eigenvectors v_1 , $-v_2$ and v_3 , c.f. eq.(29) in [19]. We have corrected a misprint in the second element of the vector v_2 in [19].

5.2 Case II

Corresponding seesaw models can be easily derived. An example is (in notation of [19]):

$$\mathbf{m}_D \sim \begin{pmatrix} x^2 y & 0 & 0 \\ 0 & x & -x \\ 0 & x^2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{M}_R^{-1} \sim \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (46)$$

where \mathbf{m}_D is the Dirac mass matrix and \mathbf{M}_R^{-1} denotes inverse of the Majorana mass matrix for the righthanded neutrinos⁴. Then

$$\tilde{N}_{II} = U_+ M_{L,II} U_+^T = \mathbf{m}_D^T \mathbf{M}_R^{-1} \mathbf{m}_D = \sqrt{\Delta m_{atm}^2} \begin{pmatrix} 2\epsilon & 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & 0 & 0 \end{pmatrix}, \quad (47)$$

with the eigenvalues

$$\lambda_1 = -\sqrt{1+\epsilon^2} + \epsilon, \quad \lambda_2 = \sqrt{1+\epsilon^2} + \epsilon, \quad \lambda_3 = 0 \quad (48)$$

and

$$|\epsilon| = 2^{-3/2} xy |a_{11}/a_{12}| = \Delta m_\odot^2 / (4\Delta m_{atm}^2) = 3.4 \times 10^{-8}. \quad (49)$$

Evidently corrections of order ϵ to U_{MNS} are negligibly small. The spectrum of light neutrinos contains two heavier states. The electron neutrino is a nearly maximal mixture of these states with a tiny admixture of the third state which is massless. We call this spectrum *semi-degenerate* ($m_1 \approx m_2 \gg m_3$). The ratio $|a_{11}/a_{12}|$ must be very small which implies that the two heavy Majorana neutrinos form a pseudo-Dirac system[21]. In our opinion the semi-degenerate mass pattern is more attractive for solutions of the solar neutrino problem with larger values of $\Delta m_\odot^2 / \Delta m_{atm}^2$ [18], see also discussion in [24].

5.3 Case III

As in case *II* the parameter ϵ is tiny and the related corrections are negligible.

5.4 Case IV

For *degenerate* neutrino masses[22, 23] one can choose the eigenvalues λ_i in the following way:

$$\lambda_1 = -1 + \epsilon, \quad \lambda_2 = 1 + \epsilon, \quad \lambda_3 = \eta + \xi, \quad (50)$$

with

$$|\epsilon| = \Delta m_\odot^2 / (4m^2) \quad \text{and} \quad |\xi| = \Delta m_{atm}^2 / (2m^2). \quad (51)$$

Corrections to eq.(38) are small for $|\xi| \ll |s|$.

⁴ This form of \mathbf{M}_R^{-1} means that either one of the righthanded neutrinos is much heavier and decoupled or there are only two heavy neutrinos in the particle spectrum and their couplings to the lefthanded neutrinos are given by the matrix

$$\mathbf{m}_D \sim \begin{pmatrix} xy & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

If the case of degenerate neutrino masses is a reasonable option for vacuum oscillations remains an open question. On one side dynamical models have been recently found [26] for the degenerate mass pattern. On the other side the parameter ϵ is extremely small and the problem of stability may be very serious, see [44]. It is also not clear if neutrino masses in the range of 1 eV are really needed by cosmology, see e.g. [45]. Fortunately it is exactly this mass range and mass pattern for which our estimations give numbers close to the present experimental limit for $0\nu 2\beta$ decays. Therefore one may hope that a definite answer will come from the experiment in not very distant future.

6 Summary

Neutrino oscillations and Majorana masses are discussed assuming bimaximal mixing of leptons. In this scheme the atmospheric neutrino anomaly [3] is described as a result of $\nu_\mu \rightarrow \nu_\tau$ oscillations and deficits of solar neutrinos [4] as vacuum oscillations of ν_e . A simple hypothesis is proposed that deviations from bimaximal mixing are dominated by a rotation in the electron–muon plane of the charged lepton sector $I_3 = -1/2$. This hypothesis is in agreement with the present data and can be tested by future precision data on solar neutrinos. Predictions for oscillations in long baseline experiments are given. There are four classes of neutrino mass patterns leading to quite different predictions for the rates of neutrinoless double beta decays.

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